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# FEYNMAN RULES IN THE TYPE III NATURAL\* FLAVOUR-CONSERVING TWO-HIGGS DOUBLET MODEL

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## ABSTRACT

We consider a two Higgs-doublet model with  $S_3$  symmetry, which implies a  $\frac{\pi}{2}$  rather than 0 relative phase between the vacuum expectation values  $\langle \Phi_1 \rangle$  and  $\langle \Phi_2 \rangle$ . The corresponding Feynman rules are derived accordingly and the transformation of the Higgs fields from the weak to the mass eigenstates includes not only an angle rotation but also a phase transformation. In this model, both doublets couple to the same type of fermions and the flavour-changing neutral currents are naturally suppressed. We also demonstrate that the Type III natural flavour-conserving model is valid at tree-level even when an explicit  $S_3$  symmetry breaking perturbation is introduced to get a reasonable CKM matrix. In the special case  $\beta = \alpha$ , as the ratio  $\tan \beta = \frac{v_2}{v_1}$  runs from 0 to  $\infty$ , the dominant Yukawa coupling will change from the first two generations to the third generation. In the Feynman rules, we also find that the charged Higgs currents are explicitly left-right asymmetric. The ratios between the left- and right-handed currents for the quarks in the same generations are estimated.

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# 1. INTRODUCTION

The two-Higgs doublet model<sup>1–8,13,14</sup> is a rather popular extension of the standard model. Introducing another doublet would lead to the flavor-changing neutral current<sup>4–8,11</sup> (FCNC) problem in its Yukawa coupling sector. There are two ways to avoid this problem, which<sup>11</sup> are generally called the two types of natural flavour conserving (NFC) models. One<sup>4–6,11</sup> is to let only one Higgs doublet couple to the fermions and the other to let the doublets couple separately to different types of fermions. In our previous investigations<sup>7–10</sup>, we found another way in which both doublets can couple to all types of fermions without FCNC by introducing  $S_3$  permutation symmetry. In that model, both doublets couple to all types of fermions and the relative phase  $\theta$  between the vacuum expectation values (VEV's) of the doublets is found to be  $\frac{\pi}{2}$ . The coupling constant matrices corresponding to the doublets can be diagonalized simultaneously by the same transformation so that the FCNC problem was naturally solved in the  $S_3$  model.

In the two-Higgs doublet models, the relative phase  $\theta$  between the VEV's could be either 0 or  $\frac{\pi}{2}$ . The  $\theta=0$  case has been considered by Bertolini<sup>5</sup>. Since the  $S_3$  model implies  $\theta = \frac{\pi}{2}$ , our main interest here is to investigate this case, where we have a pure imaginary vacuum expectation value (VEV) for the second Higgs doublet  $\Phi_2$ . The transformation from the weak to the mass eigenstates of the Higgs fields will be altered considerably by the imaginary property of  $\langle \Phi_2 \rangle$ . The detail is given in section 2.

In most two-Higgs doublet models, the Yukawa couplings of the Higgs doublets to the fermions can not be diagonalized simultaneously, which lead to the FCNC problem. We present another natural flavour-conserving (NFC) model with explicit  $S_3$  symmetry breaking in section 3. The CKM matrix is also discussed there.

Since  $\langle \Phi_2 \rangle$  becomes imaginary in the  $S_3$  model, the Feynman rules in ref. [5] should be modified. The derivation is given in section 4. Since one can couple both doublets to the same type of fermions without FCNC, the derived Feynman rules contain contributions from both doublets and are very different from those

given before. We also find that the dominant contribution to the Feynman rules will change from the first (lighter) two generations to the third (heavier) generation of fermions as the ratio  $\frac{v_2}{v_1} = \tan \beta$  varies from 0 to  $\infty$ . We conclude this paper in section 5.

## 2. THE $\theta = \frac{\pi}{2}$ CASE IN THE TWO-HIGGS DOUBLET MODEL

The mass spectra and physical eigenstates of Higgs fields have been illustrated in ref. [5] on a simplified two-Higgs doublet model which takes the relative phase  $\theta = 0$ . In general, one needs to introduce an additional symmetry under which the doublets transform differently so as to distinguish the doublets. The most popular transformation is  $\Phi_1 \rightarrow \Phi_1$ ,  $\Phi_2 \rightarrow -\Phi_2$  and the corresponding general  $SU(2)_L \times U(1)_Y$  gauge-invariant, renormalizable Higgs potential is given by *Bertolini*<sup>5</sup> and *Toussaint*<sup>12</sup>.

In breaking the original  $SU(2)_L \times U(1)_Y$  gauge symmetry down to  $U(1)_{EM}$ , the VEV's are chosen as  $\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}}$  and  $\langle \Phi_2 \rangle = \frac{v_2 e^{i\theta}}{\sqrt{2}}$ , where  $\theta$  is the phase difference between the vacuum expectation values. After SSB, the Higgs fields are written as

$$\Phi_1 = \begin{pmatrix} \frac{H_{1c}}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}(v_1 + R_1 + iI_1) \end{pmatrix}, \Phi_2 = \begin{pmatrix} \frac{H_{2c}}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}(iv_2 + R_2 + iI_2) \end{pmatrix}. \quad (2.1)$$

where  $H_{ic}$ 's are the charged Higgs fields and  $R_i$ 's ( $I_i$ 's) are the real (imaginary) parts of the neutral Higgs fields. The minimization condition  $\frac{\partial V}{\partial \theta} = 0$  tells us that  $\theta$  can only be 0 or  $\frac{\pi}{2}$ . The case  $\theta=0$  was discussed in ref. [5]. In this paper, we shall concentrate on the case  $\theta = \frac{\pi}{2}$ , which change the sign of  $\lambda_5$  in ref. [5] and mix the real and imaginary parts of the neutral Higgs fields. Here is the summary:

(a). The charged components of  $\Phi_1$  and  $\Phi_2$  mix to give a charged Goldstone boson  $G^+$  and a physical charged Higgs  $H^+$ . The transformation U from their weak eigenstates to mass eigenstates includes not only an angle rotation but also

a phase transformation as follows

$$U = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \quad (2.2)$$

where  $\tan\beta = \frac{v_2}{v_1}$ .

(b). For  $\theta = \frac{\pi}{2}$ , the constructions of the neutral Higgs fields are different to that for  $\theta = 0$ . In the  $\theta = 0$  case,  $R_1$  and  $R_2$  mix to give the scalar fields and  $I_1, I_2$  mix to give the pseudoscalar ones. But, in the  $\theta = \frac{\pi}{2}$  case,  $R_1$  and  $I_2$  are combined to give the scalar fields while  $I_1$  and  $R_2$  are combined to give the pseudoscalars. The real part and the imaginary part mix together. Therefore, the transformation between the weak and mass eigenstates also need an additional phase transformation like that in (2.2).

For pseudoscalar fields,  $I_1$  and  $-iR_2$  mix to give a Goldstone boson  $G^0$  and a physical pseudoscalar  $H_3^0$  with the same  $U$  as in the charged Higgs fields. For the scalar fields,  $R_1$  and  $iI_2$  mix to give the scalar fields  $H_1^0$  and  $H_2^0$  with the transformation

$$\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} R_1 \\ iI_2 \end{pmatrix} \quad (2.3)$$

where  $\tan\alpha = \frac{-(A-B)+\sqrt{(A-B)^2+C^2}}{C}$  with A, B and C defined in ref. [5].

In the former diagonalization of the Higgs fields, it is found that the charged Higgs and the pseudoscalars have the same mixing angle. But the mixing angle for the neutral scalars is different. One may define a set of new doublets  $\Phi'_1$  and  $\Phi'_2$  in which the charged Higgs fields and the pseudoscalars are in their mass eigenstates while the scalars  $\phi_1^0, \phi_2^0$  are not.

$$\Phi'_1 = \Phi_1 \cos\beta - i\Phi_2 \sin\beta = \begin{pmatrix} G^+ \\ v + \frac{1}{\sqrt{2}}(\phi_1^0 + iG^0) \end{pmatrix},$$

$$\Phi'_2 = -\Phi_1 \sin\beta - i\Phi_2 \cos\beta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\phi_2^0 + iH_3^0) \end{pmatrix}. \quad (2.4)$$

where  $v = \frac{1}{2}(v_1^2 + v_2^2)$ . The relation between  $H_1^0$ ,  $H_2^0$  and  $\phi_1^0$ ,  $\phi_2^0$  is

$$\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \cos(\alpha - \beta) & \sin(\alpha - \beta) \\ -\sin(\alpha - \beta) & \cos(\alpha - \beta) \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \quad (2.5)$$

If  $\beta = \alpha$ , then all the Higgs fields are diagonalized simultaneously by the transformation (2.2).

### 3. THE $S_3$ MODEL AND ITS FCNC

In standard model, there is only one Higgs doublet whose phase of VEV can be rotated away by a gauge transformation. Therefore, no spontaneous CP-violation will appear in standard model. It was widely suggested that one needs at least one more Higgs doublet to produce CP-violation spontaneously. But, most two-doublet models meet the FCNC problem which arises from the non-simultaneous diagonalization of the coupling constant matrices those couple to different doublets respectively. Glashow and Weinberg<sup>11</sup> suggested two ways to *avoid* the FCNC problem. One is to let the doublets couple separately to different types of fermions. The other is to let only one doublet couples to both types of fermions. In our previous investigations<sup>7-10</sup>, we found another way to suppress the FCNC naturally with an additional  $S_3$  permutation symmetry.

The motivation of introducing the  $S_3$  symmetry is based on the similarity between the three generations of fermions. For the same type of fermions, they are very similar to each other except their masses. It is reasonable to assume that there is no fermion masses before SSB and thus no difference between the generations. For three generations of fermions,  $S_3$  symmetry is conserved before SSB. When SSB happen, the fermions get their masses and the  $S_3$  symmetry was *spontaneously* broken. In our  $S_3$  model, the fermions are classified to the three-dimensional

representations<sup>8</sup>  $\Gamma^6$  of  $S_3$ , which is just the change of generations. The Yukawa Lagrangian with both doublets coupled to the down-type right-handed fermions is then written as

$$\begin{aligned} L_{YUK}^d &= \bar{Q}'_L (\Phi_1 G_1 + \Phi_2 G_2) D'_R + H.C. \\ &= (\bar{U}V, \bar{D})_L [\Phi'_1 (cG'_1 + isG'_2) + \Phi'_2 (-sG'_1 + icG'_2)] D_R + H.C. \end{aligned} \quad (3.1)$$

where  $Q'_L = (U', D')_L$  means the left-handed quark fields in the weak eigenstates and  $V$  is the CKM-matrix. The unprimed  $Q_L$ ,  $U_L$  and  $D_L$  are in their mass eigenstates. The unprimed  $G_1$ ,  $G_2$  are the Yukawa coupling constant matrices for the down type quarks and couple to  $\Phi_1$  and  $\Phi_2$ , respectively. The primed coupling constant matrices are defined as  $G'_i = U_d G_i U_d^\dagger$  that are diagonal. The primed Higgs doublets is defined in (2.4) and  $c = \cos \beta$ ,  $s = \sin \beta$  for simplicity. The mass matrix of the down type quarks under the spontaneous  $S_3$  breaking model is expressed as

$$\begin{aligned} M_d &= M_{d1} + M_{d2} = \langle \Phi_1 \rangle G_1 + \langle \Phi_2 \rangle G_2 \\ &= \langle \Phi_1 \rangle \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix} + \langle \Phi_2 \rangle \begin{pmatrix} 0 & -d & d \\ d & 0 & -d \\ -d & d & 0 \end{pmatrix}, \\ &= \begin{pmatrix} A & B - iD & B + iD \\ B + iD & A & B - iD \\ B - iD & B + iD & A \end{pmatrix}, \end{aligned} \quad (3.2)$$

where  $A = \frac{av_1}{\sqrt{2}}$ ,  $B = \frac{bv_1}{\sqrt{2}}$  and  $D = \frac{dv_2}{\sqrt{2}}$  and the phase of  $\Phi_2$  must be  $\pi/2$ . Since  $G_1$  and  $G_2$  can be diagonalized simultaneously by the same transformation matrix. The FCNC problem does not appear at tree-level.

The matrix form of the up type quarks are similar to (3.2) with  $A$ ,  $B$  and  $D$  replaced by  $A'$ ,  $B'$  and  $D'$ . The eigenvalues (quark masses) are given as follows

$$\begin{aligned} m_d &= A - B - \sqrt{3}D, & m_s &= A - B + \sqrt{3}D, & m_b &= A + 2B \\ m_u &= A' - B' - \sqrt{3}D', & m_c &= A' - B' + \sqrt{3}D', & m_t &= A' + 2B' \end{aligned} \quad (3.3)$$

Substituting the experimental current quark masses into these expressions and

taking the unknown top quark mass to be 135 GeV, we obtain the values of the parameters as follows

$$\begin{aligned} A &= 1.828 \text{ GeV}, & B &= 1.736 \text{ GeV}, & D &= 0.048 \text{ GeV}, \\ A' &= 45.45 \text{ GeV}, & B' &= 44.77 \text{ GeV}, & D' &= 0.388 \text{ GeV} \end{aligned} \quad (3.4)$$

where we have used the quark masses data:  $m_u = 0.0051$ ,  $m_d = 0.0089$ ,  $m_s = 0.175$ ,  $m_c = 1.35$  and  $m_b = 5.2$  in GeV.

After SSB, the Yukawa Lagrangian for the down type right-handed quarks is given by

$$\begin{aligned} L_{Yuk}^d &= (\bar{U}V, \bar{D})_L \\ &\times \left( \begin{aligned} &\frac{gM_d^d}{\sqrt{2}M_W}G^+ + \frac{g}{\sqrt{2}M_W}(-M_{d1}^d \tan \beta + M_{d2}^d \cot \beta)H^+ \\ &M_d^d + \frac{gM_d^d}{2M_W}(\phi_1^0 + iG^0) + \frac{g}{2M_W}(-M_{d1}^d \tan \beta + M_{d2}^d \cot \beta)(\phi_2^0 + iH_3^0) \end{aligned} \right) \\ &\times D_R + H.C. \end{aligned} \quad (3.5)$$

where the superscript d on the mass matrices means diagonalized. The Yukawa Lagrangian for the up type quarks can be derived in a similar way.

In the above discussions, the mass matrices corresponding to different Higgs doublets are diagonalized by the same unitary transformation and thus no FCNC problem at all. But it also leads to a  $3 \times 3$  unit CKM matrix. In ref. [8], we added an explicit  $S_3$  breaking  $P'$  to the up type quarks as a perturbation of the originally spontaneously broken  $M_u^{(0)}$  to get a reasonable CKM matrix. Since we did not add any perturbation to  $M_d^{(0)}$ , the neutral currents of down type quarks are still flavour-conserving at tree level. In what follows, we shall demonstrate that the neutral currents of up type quarks is also NFC at tree level if the perturbation is chosen suitably.

We devide the perturbation  $P'$  into  $P'_1$  and  $P'_2$  which correspond to the doublets respectively. The mass matrices are defined as follows

$$M_u = M_u^0 + P' = M_{u1}^0 + P'_1 + M_{u2}^0 + P'_2, \quad M_d = M_d^0 = M_{d1}^0 + M_{d2}^0 \quad (3.6)$$

where  $M^0$  are the spontaneous  $S_3$  breaking matrices and  $P'_i$ 's are the perturbations.

The matrix  $M_u$  is diagonalized in two stages. In the first stage,  $M_{d1}^0$ ,  $M_{d2}^0$ ,  $M_{u1}^0$  and  $M_{u2}^0$  are simultaneously diagonalized by  $U^{(0)}$  and  $P'_i$  transform to  $P_i$ .

$$U^{(0)}M_uU^{(0)\dagger} = D_u^0 + P = D_{u1}^0 + P_1 + D_{u2}^0 + P_2$$

$$U^{(0)}M_dU^{(0)\dagger} = D_d^0 = M_{d1}^d + M_{d2}^d = \text{Diad.}(m_d, m_s, m_b) \quad (3.7)$$

The matrix  $D_u^0 + P$  is then diagonalized by  $U^{(n \geq 1)} = V$  in the second stage

$$M_u^{diag} = V(D_u^0 + P)V^\dagger = (M'_{u1} + M_{u1}^P) + (M'_{u2} + M_{u2}^P) = \text{Diag.}(m_u, m_c, m_t) \quad (3.8)$$

where  $U^{(n)}$  is the correction up to the n'th order in the perturbation.

The Yukawa couplings of  $H_1^0$  and  $G^0$  are proportional to  $M_u^{diag}$  so that the corresponding neutral currents are NFC while the couplings of  $H_2^0$  and  $H_3^0$  are proportional to

$$-\frac{\sin \alpha}{\cos \beta}(M'_{u1} + M_{u1}^P) + \frac{\cos \alpha}{\sin \beta}(M'_{u2} + M_{u2}^P) \quad (3.9)$$

which should be diagonal for the sake of NFC. Since both (3.8) and (3.9) are diagonal,  $(M'_{u1} + M_{u1}^P)$  and  $(M'_{u2} + M_{u2}^P)$  must also be both diagonal. Since  $D_{u1}^0$  and  $D_{u2}^0$  are known and  $U^{(n)} = V$  is calculated in the previous investigation<sup>8</sup>, we can calculate  $M'_{u1}$  and  $M'_{u2}$  whose off-diagonal elements are just canceled by those of  $M_{u1}^P$  and  $M_{u2}^P$  respectively. Therefore, the simplest choice of  $M_{u1}^P$  and  $M_{u2}^P$  is

$$M_{u1}^P = \begin{pmatrix} 0 & 0 & -E^*3B' \\ 0 & 0 & -F^*3B' \\ -E3B' & -F3B' & 0 \end{pmatrix}$$

$$M_{u2}^P = \begin{pmatrix} 0 & -2D^*\sqrt{3}D' & -E^*\sqrt{3}D' \\ -2D\sqrt{3}D' & 0 & F^*\sqrt{3}D' \\ -E\sqrt{3}D' & F\sqrt{3}D' & 0 \end{pmatrix} \quad (3.10)$$

where, for simplicity, we use the first order result  $U^{(1)} = V$ . The inverse transformations  $V^\dagger M_{ui}^P V$  then give the required perturbations  $P_i$  which give NFC neutral



currents at tree level. Thus, we can always choose explicit  $S_3$  symmetry breaking perturbation which preserves NFC at tree level and at the same time produces reasonable CKM matrix.

#### 4. THE MODIFIED FEYNMAN RULES

In this section, we derive the Feynman rules corresponding to the phase difference  $\theta = \frac{\pi}{2}$  between the vacuum expectation values of the Higgs fields. We find that the vertices involving the Higgs boson-gauge boson trilinear interactions and the Higgs boson-gauge boson four-point interactions are not modified. Only vertices involving the fermion Yukawa coupling are modified. The relevant modified Feynman rules of the quark Yukawa couplings in the 't Hooft-Feynman gauge are

$$\begin{aligned}
& \frac{-ig}{2M_W} (M_{d2}^d \frac{\sin\alpha}{\sin\beta} + M_{d1}^d \frac{\cos\alpha}{\cos\beta}) \\
& \frac{-ig}{2M_W} (M_{d2}^d \frac{\cos\alpha}{\sin\beta} - M_{d1}^d \frac{\sin\alpha}{\cos\beta}) \\
& \frac{g\gamma_5}{2M_W} (M_{d2}^d \cot\beta - M_{d1}^d \tan\beta) \\
& \frac{g\gamma_5}{2M_W} M_D^d \\
& \frac{-ig}{2M_W} (M_{u2}^d \frac{\sin\alpha}{\sin\beta} + M_{u1}^d \frac{\cos\alpha}{\cos\beta}) \\
& \frac{-ig}{2M_W} (M_{u2}^d \frac{\cos\alpha}{\sin\beta} - M_{u1}^d \frac{\sin\alpha}{\cos\beta}) \\
& \frac{g\gamma_5}{2M_W} (M_{u2}^d \cot\beta - M_{u1}^d \tan\beta) \\
& \frac{-g\gamma_5}{2M_W} M_U^d \\
& \frac{-igV_{ij}}{2\sqrt{2}M_W} [(1 - \gamma_5)(M_{u1}^d \tan\beta - M_{u2}^d \cot\beta)_{ii} + (1 + \gamma_5)(M_{d2}^d \cot\beta - M_{d1}^d \tan\beta)_{jj}] \\
& \frac{-igV_{ji}^{-1}}{2\sqrt{2}M_W} [(1 + \gamma_5)(M_{u1}^d \tan\beta - M_{u2}^d \cot\beta)_{ii} + (1 - \gamma_5)(M_{d2}^d \cot\beta - M_{d1}^d \tan\beta)_{jj}]
\end{aligned}$$

We note that the quark Yukawa vertices corresponding to the Goldstone bosons are the same as those in ref. [5], but the vertices corresponding to the physical Higgs bosons are considerably different. We also note that the vertices of the charged Higgs fields depend only on  $\beta$ .

In the case of  $\beta = \alpha$ , all Higgs fields are diagonalized simultaneously. This leads to the following interesting results: The  $H_1^0$  vertices reduce to those given in ref. [5], while those vertices corresponding to  $H_2^0$ ,  $H_3^0$  and  $H^\pm$  become

$$\begin{aligned} & \frac{-ig}{2M_W}(M_{d2}^d \cot \beta - M_{d1}^d \tan \beta)_{jj} \\ & \frac{g\gamma_5}{2M_W}(M_{d2}^d \cot \beta - M_{d1}^d \tan \beta)_{jj} \\ & \frac{-ig}{2M_W}(M_{u2}^d \cot \beta - M_{u1}^d \tan \beta)_{ii} \\ & \frac{-g\gamma_5}{2M_W}(M_{u2}^d \cot \beta - M_{u1}^d \tan \beta)_{ii} \end{aligned}$$

We observe that in the above vertices, all terms depend on  $X_i = (M_{u1}^d \tan \beta - M_{u2}^d \cot \beta)_{ii}$  or  $Y_j = (M_{d1}^d \tan \beta - M_{d2}^d \cot \beta)_{jj}$ . The  $X_i$  terms always appear in the vertices involving the up-type quarks, while the  $Y_j$  terms always appear in those involving the down-type quarks.

In the natural flavour-conserving  $S_3$  model, since  $m_u$  and  $m_d$  are too small compared with other quarks, we may assume them to be zero, which lead to  $A-B = \sqrt{3}D = x$  and  $A' - B' = \sqrt{3}D' = x'$ . Then we may express  $X$  and  $Y$  as follows

$$\begin{aligned} X = M_{u1}^d \tan \beta - M_{u2}^d \cot \beta &= \begin{pmatrix} x'(\tan \beta + \cot \beta) & 0 & 0 \\ 0 & x'(\tan \beta - \cot \beta) & 0 \\ 0 & 0 & z' \tan \beta \end{pmatrix}, \\ Y = M_{d1}^d \tan \beta - M_{d2}^d \cot \beta &= \begin{pmatrix} x(\tan \beta + \cot \beta) & 0 & 0 \\ 0 & x(\tan \beta - \cot \beta) & 0 \\ 0 & 0 & z \tan \beta \end{pmatrix}. \end{aligned} \tag{4.1}$$

where  $z = A + 2B$  and  $z' = A' + 2B'$ .

For the neutral  $H_2^0$  and  $H_3^0$  vertices, the relative magnitudes of the vertices depend only on those of the diagonal elements of  $X$  and  $Y$ . One may find in (4.1) that  $\tan \beta$  dominates the variations of the elements. We discuss only the following three special cases here and the details are shown in Fig. (1) to (4).

(1).  $\tan \beta \rightarrow 0$ : The couplings for the top and bottom quarks are very small. The contributions only come from the first two generations. This contradicts the general assumption of top and bottom dominance. We also note that the vertex factors for the first two generations are of different signs.

(2).  $\tan \beta \rightarrow \infty$ : The top and bottom vertices dominate over the lighter ones by about  $\frac{z'}{x'} \sim 200$  times for the up type and  $\frac{z}{x} \sim 60$  times for the down type, so we may neglect the first two generations. This agrees the general assumption of heavy quark dominance.

(3).  $\tan \beta = O(10^{-1})$ : Assuming that the couplings of the first two generations are of the same order of magnitude as those of the third generation, then  $\tan^2 \beta \sim \frac{x}{z} (\frac{x'}{z'})$  for the down (up) type quarks. When  $\tan \beta$  is of the order  $10^{-1}$ , no vertex should be neglected.

There are also something interesting in the charged Higgs mediated currents. Since  $X_i$  and  $Y_j$  varies as  $\beta$  changes from 0 to  $\frac{\pi}{2}$ , the relative magnitudes of the left- and right-handed charged currents may also change. For  $i=j$ , the ratio of the left-right currents in the  $H^+$  vertices is about 7 to 1 for the first two generations and about 27 to 1 for the third generation. These ratios would be reversed in the  $H^-$  vertices. When  $i \neq j$ , the ratios of the left- and right-handed currents depend on  $\tan \beta$  and can not be determined.

## 5. CONCLUSIONS AND DISCUSSIONS

In this paper, we present and discuss the Feynman rules for the relative phase  $\theta = \frac{\pi}{2}$  instead of the generally considered  $\theta = 0$ . The roles of the Higgs fields are very different from those for  $\theta = 0$ . This non-vanishing  $\theta$  leads to a pure imaginary VEV for  $\Phi_2$  and the real and imaginary parts of the neutral Higgs mix to give the mass eigenstates, i.e.  $(R_1, I_2) \rightarrow (H_1^0, H_2^0)$  and  $(I_1, R_2) \rightarrow (G^0, H_3^0)$ . One needs additional phase transformation to diagonalize the Higgs fields, which is absent in the  $\theta = 0$  case. We also present a Type III NFC model with  $S_3$  symmetry which is valid at tree-level even when an explicit  $S_3$  breaking perturbation is introduced to get a reasonable CKM matrix.

In the special case  $\beta = \alpha$ , we find that the generally-considered heavy-quark dominance is valid only when  $\tan\beta$  is larger than 0.075 (0.13) for the up (down) type quarks. When  $\tan\beta$  is smaller than these values, the first two generations dominate over the third generation. The variations of  $X_i$  ( $Y_j$ ) in the Yukawa couplings of the up (down) type quarks are shown in Fig. (1) and (2) (Fig. (3) and (4)) as functions of  $\beta$ . We also noticed the left-right asymmetry in the charged currents mediated by  $H^\pm$ . Independent of the ratio  $\tan\beta$  between  $v_2$  and  $v_1$ , the  $i=j$  vertices are clearly left-right asymmetric. The ratios of the left-right asymmetry in the  $H^+$  mediated charged currents are estimated to be about 7 to 1 for the first two generations and about 27 to 1 for the third generation. The ratios in the  $H^-$  mediated currents are reverse to those mentioned above. For  $i \neq j$ , the currents cannot be adjusted to be left-right symmetric simultaneously, and so there must be left-right asymmetry in the charged Higgs mediated currents at tree level.

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### Figure Captions

Fig. (1). The  $X_i$  factor in the Yukawa couplings of the up-type quarks with  $H_2^0$  and  $H_3^0$  as  $\beta$  varies from 0 to  $\frac{\pi}{2}$ . The lines of the u and c quarks are so close that they overlap.

Fig. (2). The enlarged part of the intersecting region in Fig. (1). The light quarks dominate for  $\beta < 0.075$ .

Fig. (3). The  $Y_j$  factor in the Yukawa couplings of the up-type quarks with  $H_2^0$  and  $H_3^0$  as  $\beta$  varies from 0 to  $\frac{\pi}{2}$ . The lines of the d and s quarks are so close that they overlap.

Fig. (4). The enlarged part of the intersecting region in Fig. (3). The light quarks dominate for  $\beta < 0.13$ .

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